

The Basis of Jewish Mathematical Education

Excerpt from an upcoming volume on Torah and Mathematics

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Numbers and Symbols

On Passover eve, just before the conclusion of the *Seder*, the custom of many families is to recite or sing the ancient poem titled, “Who Knows One?” (מִי יוֹדֵד אֶת אֶתְּ). The poem appears in almost all texts of the *Hagadah*, the text recited on this special night. It is structured in a question and answer format.

The poem begins,

Who knows one? I know one: One is our God in heaven and on earth.

In this format, the poem continues, with each stanza pondering the Jewish significance of the numbers (the integers) from 1 to 13.

The *gematria* of “one” (אֶתְּ) is 13. So in essence, the poem starts with 1 and ends with “one” (אֶתְּ).¹ The answer provided in the poem identifies each integer with a well-known Jewish symbol or concept.

It is well known that the *Seder* is one of the rituals performed in practice by almost all Jews, regardless of level of religious adherence. Because of the central place occupied by the *Seder* in Jewish ritual life, it would be fitting to say that the manner in which this poem treats numbers serves in effect as the foundational example for Jewish mathematical education. Following the poem’s content, we may state that the foundation of Jewish mathematical education is forging an association between numbers and particular Jewish concepts they relate to. The child is taught that every number is related directly to a Jewish symbol, in such a way that mere mention of the number will bring up its associated symbol in the mind.

Numbers and the Exodus

We might ask, why does this poem appear in the *Hagadah* that is devoted to the Exodus from Egypt, in the first place? To answer this question we must begin by noting that the *Seder* night is *the* time par excellence devoted to Jewish family education. The Torah commands all parents to educate their children about the Exodus from Egypt,

And you shall tell your son on that day, saying, "In merit of this, God acted on my behalf in Egypt."²

Education is so essential to the Seder night that educating our children on this night is considered one of the Torah's 613 commandments. So, one might conclude that somehow, educating children about numbers is related to telling the story of the Exodus from Egypt, which should include as much as possible. Indeed, the sages who compiled the *Hagadah* tell us that, "the more one tells the story of the Exodus from Egypt, the more praiseworthy."

At the time of the Exodus, Egypt was the world-superpower, the center of human knowledge and science. When God foretold Abraham about the exile in Egypt, He also promised him that his children would leave with great treasures. Much of this treasure was physical (representing back-pay and reparations for 210 years of forced slavery). But part of it was more conceptual and included what at the time was the most advanced knowledge of science in mathematics.

The purpose of the Exodus was ultimately God's revelation and the giving of the Torah at Mt. Sinai. Thus, the Exodus cannot be understood as merely the freeing of the Jewish people from their physical bondage in Egypt. Rather, the exodus represents the first step in liberating all of mankind, and all of mankind's knowledge, from being trapped in a purely mundane reality. The Torah's purpose is to elevate mankind's spirit by reconnecting us all with the Creator, who wills that the spiritual and the mundane exist as one.

A most important step in elevating our knowledge and science is achieved by revealing the deep Divine meanings of numbers. The sages stress that numbers are the building blocks of our reality.³ This statement goes far beyond Galileo's famous assertion that, "Nature is a book and the book's language is mathematics."

Indeed, to stress the importance of numbers and the need to sanctify their meaning and usage, we find that the first commandment God gave the Jewish people, while they were still in Egypt, was the commandment of sanctifying the new moon. In other words, establishing the Jewish calendar is the very basis of Jewish practice. This commandment, more than any other, requires a deep understanding of the interplay between numbers and the laws of nature.

So, the first thing we learn from the inclusion of this poem "Who Knows One?" in the *Hagadah* is that establishing a proper relationship with numbers in particular and mathematics in general, is a cornerstone of the Torah's educational program. Later, we will cover the basic associations between numbers and concepts found in this poem and in the Torah in general. For now, we will turn to analyze the poem's structure. We will see that not only are its contents aimed at developing a Torah-based understanding of

numbers and their meaning, but its form too reflects some of the most basic patterns used to analyze texts mathematically.

Who Knows [the Meaning of] “One?”

The first stanza reads,

Who knows [the meaning of] one? I know [the meaning of] one! One is our God in heaven and on earth.

And in the original Hebrew,

אֶחָד מִי יוֹדֵעַ? אֶחָד אֲנִי יוֹדֵעַ! אֶחָד אֱלֹהֵינוּ שֶׁבְּשָׁמַיִם וּבָאָרֶץ

[Because the addition of the words “the meaning of” is implied throughout the poem, from now on we will refrain from adding it when quoting from the poem. The reader should simply assume its presence.]

Counting, we find that the first stanza has 10 words and 39 letters. 39 is of course a multiple of 13, the *gematria* of “one” (אֶחָד),

$$39 = 3 \cdot \text{אֶחָד} = 3 \cdot 13$$

But, this is more than just an allusion to this being the stanza referring to the number one, because in it the word “one” (אֶחָד) itself appears 3 times!

39 is also the sum of the *gematria* of God’s essential Name, *Havayah* (יְהוָה), which equals 26 and “one” (אֶחָד), which equals 13. These two words also terminate the *Shema*, the quintessential statement of Jewish faith in the oneness of God, “Hear O’ Israel, *Havayah* is our God, *Havayah* is one” (שְׁמַע יִשְׂרָאֵל יְהוָה אֱלֹהֵינוּ יְהוָה אֶחָד). Thus, the number 39 is closely associated with “one” as it pertains to God, exactly the identity made in the first stanza.

The basic structure of 10 words (with only the number changing) found in the first stanza—“Who knows x? I know x. One is our God in Heaven and on earth”—is retained throughout the first 10 stanzas, with only the number, designated by x, changing (after which, because the number requires two words to denote, the number of words in the basic structure rises to 11). These 10 words divide naturally into 4 groups with 1, 2, 3, and 4 words respectively.

10 Words and 39 Letters

But, there is even further self-reference lurking here, which is revealed if we write the names of the numbers from 1 to 10 as they appear in the poem,⁴

אֶחָד שְׁנַיִם שְׁלוֹשָׁה אַרְבַּע חֲמִשָּׁה שֵׁשׁ שִׁבְעָה שְׁמוֹנָה תְּשָׁעָה עֶשְׂרִים

Counting, we find that there are once again 10 words and 39 letters. This time, the division of 39 into 26 and 13 appears explicitly; the numbers from 1 to 7 (אָחַד שְׁנַיִם שְׁלוֹשָׁה) (אַרְבַּע חֲמִשָּׁה שֵׁשׁ שִׁבְעָה) have 26 letters and the numbers from 8 to 10 (שְׁמוֹנָה עָשָׂר עֶשְׂרִים עֲשָׂרָה) have 13 letters.

Indeed, this very same structure—10 words with 39 letters—also appears in the verse following the *Shema*,

*And you shall love Havayah your God with all of your heart, with all of your soul, and with all of your might.*⁵

וְאָהַבְתָּ אֶת יְהוָה אֱלֹהֶיךָ בְּכָל לִבְבְּךָ וּבְכָל נַפְשְׁךָ וּבְכָל מְאֹדְךָ

And once more, we find this same structure—10 words with 39 letters—in the names of the ten *sefirot*:

Crown, wisdom, understanding, loving-kindness, might, beauty, victory, thanksgiving, foundation, and kingdom

כֶּתֶר חֲכָמָה בִּינָה חֶסֶד גְּבוּרָה תְּפִאֲרֹת נִצְחָה הוֹד יְסוֹד מְלָכוּת

The First Stanza and the Sefirot

On the structural level, we can find an even deeper similarity between the first stanza and the names of the ten *sefirot*. As taught in the introduction of the *Tikunei Zohar*, in their mature state, the *sefirot* are aligned along three axes and divided into 4 groupings with 1, 2, 3, and 4 *sefirot* respectively in each group. This is referred to as the, “Simple song, double song, triple song, and quadruple song.”⁶

single song	crown	
double song	understanding	wisdom
triple song	might	loving-kindness
	beauty	
quadruple song	acknowledgment	victory
	foundation	
	kingdom	

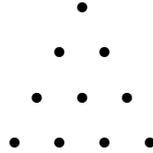
Grouping the *sefirot* in this manner provides an essential mathematical bridge between the numbers 10 and 4, where 10 is the number of the *sefirot* and 4 is the number of letters in God’s essential Name, *Havayah*; the number of Worlds in Kabbalah, etc. This simple relationship is expressed mathematically as,

$$10 = \triangle 4$$

or,

$$10 = 1 + 2 + 3 + 4$$

We illustrate the shape of the triangle as a sum of integers in the following manner,



If we structure the *sefirot* based on a triangular shape, we get,



Now, note that the 10 words of the first stanza share the same triangular structure, since the syntax divides the words naturally into groups of 1, 2, 3, and 4, as follows,



Because the *sefirot* of crown and knowledge are considered intrinsically one—i.e., knowledge is the conscious manifestation of the powers of the crown—one or the other of these two *sefirot* can be enumerated as one of the 10 *sefirot*. Note also that substituting knowledge for crown does not alter the number of letters in the names of the 10 *sefirot*, because “knowledge” (דָּעַת) and “crown” (כִּתְרֵי) both have 3 letters. However, whereas the *sefirah* of crown is the super-conscious hovering above wisdom and understanding (the intellectual *sefirot*), the *sefirah* of knowledge resides below them, acting as a synthesizing element that transfers energy from the intellect (wisdom and understanding) to the emotions (loving-kindness and below).

Thus, when substituting knowledge for crown in the three-axis depiction of the *sefirot*, we get,

double song	understanding	wisdom
single song	knowledge	
triple song	might	loving-kindness
	beauty	
quadruple song	acknowledgment	victory
	foundation	
	kingdom	

Copying the 10 words of the first stanza of “Who Knows One?” into this structure, we get,

מִי	אַחַד
יֹדֵעַ	
אָנִי	אַחַד
יֹדֵעַ	
אֱלֹהֵינוּ	אַחַד
שֶׁבַּשְׁמַיִם	
וּבְאָרֶץ	

This is by far the most mature representation of the 10 words in the first stanza of the poem. To study it, let us first copy it with the names of the *sefirot* in place,

understanding מי	wisdom אָחָד
knowledge יֹדֵעַ	
might אָנִי	loving-kindness אָחָד
beauty יֹדֵעַ	
acknowledgment אָלֵהֵינוּ	victory אָחָד
foundation שְׁבַע־מִיָּם	
kingdom וּבְאֶרֶץ	

Let us take a closer look at this correspondence.

The first thing to stand out is the three instances of “one” that populate the entire right axis. Every time the word “one” appears it represents the start of a different category of *sefirot*; from intellect (wisdom), to emotive (loving-kindness), to habitual (victory).

The word that corresponds to understanding is “who” (מי). The Zohar’s introduction states that the question “Who?” is directly related to the *sefirah* of understanding.⁷

Corresponding to knowledge is the word “knows” (יֹדֵעַ), an explicit reference.

The second instance of “knows” corresponds to the *sefirah* of beauty. Indeed, knowledge is to beauty as the soul is to the body.

Corresponding to the *sefirah* of might we find the word “I” (אָנִי), representing a state of self-consciousness. Knowing one’s own self as a distinct and separate entity is fueled by emotions of fear or distance, the inner motivator and experience of might. The Arizal writes that when first given to the newborn, both the good inclination and the so-called evil inclination are holy angels—that is instruments of holiness. The good inclination manifests as the child’s emotions of loving-kindness, while the evil inclination to emotions of might, or fear. The evil inclination’s task is to safeguard the newborn’s survival, warning it when there is some danger. It is the evil inclination for example that ensures

the child does not stick his hand into fire. Awareness of the pain causes fear of the flame. The evil inclination, as a holy angel, is also entrusted with ensuring that the child gets enough sustenance; when hunger strikes it makes the child cry for food. But, as the child matures so does the evil inclination and begins to ensure not only survival at its base level, but causes the child to demand sweets and pleasantries. Even if it has not transformed into demanding outright forbidden things and practices, through fear (of flame, of hunger, etc.) the evil inclination, by solidifying our sense of separateness through fear fuels the creation of a self-consciousness, which is recognized as the "I."⁸

To understand the connection between "our God" (אֱלֹהֵינוּ) and acknowledgment requires us to discuss the difference between "our God" a derivative of God's Name, *Elokim* (אֱלֹהִים) and God's essential Name, *Havayah*. Grammatically, the Name *Elokim* suggests plurality (it ends with the plural suffix, ים). Yet, even though it is God's essential Name, *Havayah*, that usually goes together with oneness, the poem uses the Name *Elokim*. Without a doubt, there is a reference here to the *Shema*, the statement of Jewish faith in God's oneness. The point of the *Shema* is to awaken us to meditate on the fact that even though there seem to be many powers (suggested by *Elokim*) working side-by-side in reality (both natural and human), they actually are only one. Put another way, the *Shema* is telling us that *Elokim* too, though representing the Almighty's many manifestations, is only one. Jewish meditation is particularly dependent on the intellectual faculty of understanding, which through its ability to move from general observations to very particularized ones (from general principles to specifics) is able to give birth to emotive states, corresponding to the sefirot from loving-kindness to kingdom.⁹ Since understanding is considered akin to hearing, the first word of the *Shema*, which means "Hear," corresponds to understanding. It is an invitation to meditate. The next word, "Israel" refers to the sefirah of beauty (*tiferet*), the central measure of the heart's emotive faculties. The final 4 words, "*Havayah* is our God, *Havayah* is one" (יהוה אֱלֹהֵינוּ יהוה אחד) then correspond in order to the remaining emotive faculties, from victory to kingdom, which means that the word "our God" (אֱלֹהֵינוּ) corresponds to acknowledgment.

Most beautifully, the final two words of the first stanza, "in the heavens" (שָׁבַיְמִים) and "on the earth" (וּבְאֶרֶץ), correspond to foundation and kingdom, a very straightforward parallel that appears as early as the first mention of the names of the sefirot in King David's words, "Yours, God, is the greatness, and the might, and the beauty, and the victory, and the acknowledgment, as are all in the heavens and on the earth."¹⁰ The phrase "all in the heavens" corresponds to foundation and "on the earth" corresponds there to kingdom.

Three One's

Let us meditate further on the three instances of “one” in our correspondence. As stressed earlier, the cornerstone of mathematical analysis according to Torah is associating numbers with concepts or ideas. This is also true when encountering something of the nature of the 3 “one”s in the first stanza. In the mind of even a child attending grade school this is immediately understood as a reference to the three instances of the word “one” appearing in the Genesis account of creation and describing “one day,” “one place,” and “one flesh.”

It follows that each of these three instances contextually corresponds to the three groupings of *sefirot* (the double song, triple song, and quadruple song) we have been discussing.

The first instance appears in the verse,

*And God called the light day and He called the darkness night; There was evening and there was morning, one day.*¹¹

וַיִּקְרָא אֱלֹהִים לְאוֹר יוֹם וְלַחֹשֶׁךְ קָרָא לַיְלָה וַיְהִי עֶרֶב וַיְהִי בֹקֶר יוֹם אֶחָד.

The *Book of Formation*,¹² the first Kabblistic text, analyzes the three dimensions of space, time, and soul, using a unique perspective. Space has 6 extremities (2 in each of its three dimensions), while time and soul each have two, bringing the total number of extremities (literally, depths, in Hebrew) to 10, which of course correspond to the 10 *sefirot*. In particular, the two extremities of time are called the extreme beginning and the extreme end and correspond to wisdom and understanding. Thus, “one day” provides a temporal context for the word “one” and corresponds directly with the *sefirot* of wisdom and understanding,¹³ the intellectual *sefirot* (the double song).

The second appearance of “one” in the account of creation is in the verse,

*And God said, “Let the water under heaven be gathered unto one place and the dry land will be seen,” and it was so.*¹⁴

וַיֹּאמֶר אֱלֹהִים יִקְוּ הַמַּיִם מִתַּחַת הַשָּׁמַיִם אֶל מְקוֹם אֶחָד וַתִּרְאֶה הַיַּבֵּשׁה וַיְהִי כֵן.

This alludes to the second unit of *sefirot*, those from loving-kindness to beauty, the triple song. Continuing with the *Book of Formation*, the three dimensions of space (consisting of six extremities corresponding to the six *sefirot* from loving-kindness to foundation) begin with the *sefirot* of the triple song: loving-kindness, might, and beauty.

Apropos the relationship between space and loving-kindness, the Torah relates that Abraham, the archetypal soul of loving-kindness, “Awoke early to stand in the place he had stood earlier.”¹⁵ From this verse, the sages¹⁶ learn the importance of having personal space, halachically¹⁷ (legally), for prayer, conceptually in order to better focus his energy in all his endeavors.¹⁸

Finally, the third instance appears in the verse,

*Therefore shall man leave his father and mother to cling to his wife so that they become one flesh.*¹⁹

על בן יעזב איש את אביו ואת אמו ודבק באשתו והיו לבשר אחד.

The union of male and female is most pronounced in the relationship between foundation and kingdom, the two final *sefirot* of the quadruple song. The second strongest such coupling is found between victory and acknowledgment, described in the *Zohar* as “He is in victory, she is in acknowledgment.” Let us summarize this additional correspondence by expanding our previous chart,

double song “one day” time dimension	understanding מִי	wisdom אָחַד
single song	knowledge יִדְעַע	
triple song “one place” space dimensions	might אָנִי	loving-kindness אָחַד
	beauty יִדְעַע	
quadruple song “one flesh” soul dimension	acknowledgment אָלְהֵינִי	victory אָחַד
	foundation שְׁבַשְׁמִים	
	kingdom וּבְאֶרֶץ	

We will have more to say about these three verses in part 2 of this volume.

One as the Essence of the Poem

Let us see another way in which the essence of the poem “*Who Knows One*” points indeed to the number 1. In the wording that the poem has been transmitted to us there are a number of Aramaic words. If we translate these into Hebrew, the exact Hebrew form²⁰ includes exactly 1369 letters. 1369 is a perfect square, and specifically, $1369 = 37^2$. Let us draw the Hebrew translation in the form of a square:

$$\text{וְרוּחַ אֱלֹהִים מְרַחֵף עַל פְּנֵי הַמַּיִם} = 1369 = 37^2$$

This indicates that the awareness of the inner meaning of numbers is the ever-hovering Divine prerequisite for giving the Torah. In fact, the Torah itself is likened to water, as is in the Talmud, “‘Water’ means none other than Torah.”²⁴

Oneness in the Poem

Let us quickly look at the structure of the 12 other stanzas.

The second stanza consists of 13 words; again the value of the number “one” (אָחַד), or “love” (אַהֲבָה).

The third stanza has 15 words. With the 13 words of the second stanza, we have 28, which is the value of the back-side of the word “love” (אַהֲבָה)

$$\text{א אה אהב אהבה} = 28$$

The fourth stanza has 17 words and the fifth 20. Together they have 37 words, and we have just seen the correlation of 37 with “love.”

The total words in the second, third, fourth, and fifth stanzas are thus 65, which is also a product of 13 (multiplied by 5)!

The sixth stanza has 23 words, and 23 is the value of the back-side form of “one” (אָחַד),

$$\text{א אח אחד} = 23$$

The seventh stanza has 26 words, a multiple of 13 as well and the value of *Havayah*, God’s essential Name most representative of His essential oneness. The total number of words in the sixth and seventh stanza is thus $49 = 7^2$, indicating a state of completeness.

The eighth stanza has 29 words, the value of the front-side form of “one” (אָחַד),

$$\text{ד אחד חד ד} = 29$$

The ninth has 32 words, which is the value of the letters of “one” multiplied by one another,

$$\text{ד} \cdot \text{ח} \cdot \text{א} = 32$$

32 is also the value of “single one” (יְחִיד), another topic we will look at in greater length in part II.

The tenth stanza has 34 words, or the sum of the letters of “love” (אַהֲבָה) in triangular form,

$$\Delta \text{ה} + \Delta \text{ב} + \Delta \text{ה} + \Delta \text{א} = \Delta 1 + \Delta 5 + \Delta 2 + \Delta 5 = 1 + 15 + 3 + 15 = 34$$

The eleventh stanza has 39 words, the value of “God is one” (יְיְהוָה אֶחָד) and the product of 3 times 13, the value of “one” (אָחַד).

The twelfth stanza has 42 words, which together with the 39 words of the eleventh gives 81 or 9^2 .

The thirteenth and final stanza has 45 words, which is the triangle of 9. 45 is also the value of “naught” (מָה), the expression of Moshe Rabbeinu’s essential selflessness, which we will immediately turn to in more detail. If we now add 45 to 42 (the number of words in the twelfth stanza), together they form the phrase “without essence” (בְּלִי מָה), the secret of the infinite in Kabbalah.

Summing over all the stanzas, we find that the entire poem contains 345 words. 345 is the value of “Moshe” (מֹשֶׁה).

Up to and including the twelfth stanza, there are 300 words, $300 = \Delta 24$. The remaining 45 words in the thirteenth stanza alludes to the word “naught” (מָה), spoken by Moshe on two occasions, and indicating his sense of selflessness, “And we are naught”²⁵ (וַנְּחַנוּ מָה).

Another beautiful point here is that if we add 13, the number of numbers enumerated in the poem to 345, we get the value of “Mashiach” (מָשִׁיחַ), 358. Put another way, adding “one” (אֶחָד) to “Moshe” (מֹשֶׁה) yields “Mashiach” (מָשִׁיחַ).²⁶

Indeed, Moshe Rabbeinu is intrinsically connected with the heart of the poem’s refrain, “One is our God” (אֶחָד אֱ-לֹהֵינוּ), whose value is 115. Upon completing the public reading of a part of the Pentateuch in synagogue, the entire congregation says, “Strong, strong, and we shall be strengthened” (חֲזַק חֲזַק וְנִתְחַזַּק). This is actually an allusion to Moshe Rabbeinu himself, as the gematria of “Moshe” (מֹשֶׁה) is equal to 3 times the value of “strong” (חֲזַק), 115.²⁷ Thus, “Moshe” also equals 3 times the value of “One is our God” (אֶחָד אֱ-לֹהֵינוּ),

$$\text{מֹשֶׁה} = \text{חֲזַק חֲזַק חֲזַק} = 3 \cdot \text{אֶחָד אֱ-לֹהֵינוּ}$$

The Poem’s Multi-dimensional Structure

Taking a bird’s-eye view of the poem we see that the refrain includes a countdown to one from each number that is the subject of the stanza.

So actually, we mention much more than just 13 numbers (1 through 13). In fact, if we would like to sum all the integers mentioned in the poem, we have to sum in the following manner,

$$1 + (2 + 1) + (3 + 2 + 1) + \dots + (13 + 12 + \dots + 2 + 1)$$

This is simply the sum of the triangular numbers from 1 to 13, so we can write this in shorthand as,

$$\Delta 1 + \Delta 2 + \dots + \Delta 12 + \Delta 13$$

If we calculate the value of this expression, we will find that it is 455. But, 455 is itself a multiple of 13, the value of “one” (𐤀𐤍𐤏),

$$455 = 35 \cdot 13 = 5 \cdot \triangle 13$$

Though perhaps not readily apparent, through this simple observation, we have entered a higher level of number theory. Summing triangles represents the geometric form of the three dimensional triangle called the tetrahedron. In other words, 455 is a tetrahedral number.

We define tetrahedral numbers with the following equation,

$$f[n] = \sum_{k=1}^n \triangle_k$$

We use the symbol \triangle_n to denote the nth tetrahedral number. Basically, what this equation is saying is that the nth tetrahedral number is the sum of the triangles from 1 to n. Let’s draw the first few tetrahedral numbers:



Thus, the total numbers counted in the poem “Who Knows One?” is the 13th such number/figure whose base alone is a triangle of 13.

$$455 = \triangle_{13}$$

Tetrahedral numbers are three dimensional triangular numbers and as such they are part of an infinitely large family of higher dimensional triangular numbers, or triangles in many dimensions. Beyond the 3 dimensional triangular number, the tetrahedron, it is hard for our minds to conceive or picture what the higher dimensional triangles look like, but it is very easy to calculate their value. Let us use the symbol \triangle_n to denote the nth i-dimensional triangular number.

We can then construct the following table of i-dimensional triangular numbers:

n	1	2	3	4	5	6	7	8	9	10	11	12	13
\triangle_n	1	1	1	1	1	1	1	1	1	1	1	1	1
\triangle_n	1	2	3	4	5	6	7	8	9	10	11	12	13
\triangle_n	1	3	6	10	15	21	28	36	45	55	66	78	91

Δ_n	1	4	10	20	35	56	84	120	165	220	286	364	455
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Each number in the n th column of a row is simply the sum of the numbers in the row above it from 1 to the n th column. Thus, for example, 165 is the sum of 1 through 45 in the row above.

Now, observe the 13th column. It contains the numbers 1, 13, 91, 455. We see that all the numbers connected with the poem appear in this column, which means they can be defined as the 13th i -dimensional triangular numbers.

- $1 = \Delta_{13}$ – clearly the subject of the entire poem “Who knows One?”
- $13 = \text{“one” (אֶחָד)} = \Delta_{13}$
- $91 = \text{sum of numbers from 1 to “one” (אֶחָד), } 13 = \Delta_{13}$
- $455 = \text{total of all numbers mentioned} = \Delta_{13}$

Let us write out the next row in this table of i -dimensional triangular numbers (the 4-dimensional triangular numbers),

n	1	2	3	4	5	6	7	8	9	10	11	12	13
Δ_n	1	5	15	35	70	126	210	330	495	715	1001	1365	1820

We see that the 13th number in this case is 1820.^{xxviii} It would be a wonderfully fulfilled exercise to try and recite the poem so that the sum of its integers would come to 1820! This is left for the reader as a challenge for the *Seder* night. What is essential for our present purpose is that 1820 is the 13th 4-dimensional triangular number. But, 1820 is one of the most important numbers in the Torah, perhaps the most, as it is the exact number of times God’s essential Name, *Havayah*, appears in the Torah (the Pentateuch^{xxix}). It is also the exact number of different three-letter roots found in the entire text of the Torah. As noted above, of all the Names of God, *Havayah* reflects his unity most.

¹. This is an example of the well-known principle, “Their end is enwedged in their beginning (*Sefer Yetzirah* 1:7).
². Exodus 13:8.
³. The Book of Formation states, “With 32 wondrous pathways of wisdom, God... created the world.”
⁴. In Hebrew, every number has both masculine and feminine forms. The usage of the one or the other depends on the gender of the object being referred to.
⁵. Deuteronomy 6:5.
⁶. *Zohar* III, 219b. *Tikunei Zohar* 13 (51b).
⁷. The three questions, what, who, and why correspond to the three intellectual sefirot, wisdom, understanding, and knowledge. See *Zohar* I, 1b.

8. Self consciousness is usually associated with the sefirah of kingdom whose development depends on an influx of energy from might.
9. Because rectified emotive states are figuratively born from understanding, also known as the mother principle, the sefirot from loving-kindness to kingdom are referred to as its children.
10. 1 Chronicles 29:11.
11. Genesis 1:5.
12. *Sefer Yetzirah* 1:5.
13. Understanding here corresponds to the word “who” (מי), whose letters reverse to spell the word “sea” (ים), a contracted form of “day” (יום).
14. Genesis 1:9.
15. *Ibid.* 19:27.
16. *Berachot* 6b.
17. *Shulchan Aruch Orach Chaim* 90:19.
18. Relating loving-kindness with its source in wisdom (above it on the right axis) is the saying, “Who is wise? He who knows his place” (see *Avot* 6:6)
19. Genesis 2:24.
20. The exact Hebrew form reads,

אָחַד מִי יוֹדֵעַ. אָחַד אֲנִי יוֹדֵעַ. אָחַד אֱלֹהֵינוּ שְׁבַשְׁמִים וּבְאֲרָץ:
שְׁנַיִם מִי יוֹדֵעַ. שְׁנַיִם אֲנִי יוֹדֵעַ. שְׁנֵי לַחֹת הַבְּרִית. אָחַד אֱלֹהֵינוּ שְׁבַשְׁמִים וּבְאֲרָץ:
שְׁלֹשָׁה מִי יוֹדֵעַ. שְׁלֹשָׁה אֲנִי יוֹדֵעַ. שְׁלֹשָׁה אָבוֹת. שְׁנֵי לַחֹת הַבְּרִית. אָחַד אֱלֹהֵינוּ שְׁבַשְׁמִים וּבְאֲרָץ:
אַרְבַּע מִי יוֹדֵעַ. אַרְבַּע אֲנִי יוֹדֵעַ. אַרְבַּע אֲמָהוֹת. שְׁלֹשָׁה אָבוֹת. שְׁנֵי לַחֹת הַבְּרִית. אָחַד אֱלֹהֵינוּ שְׁבַשְׁמִים וּבְאֲרָץ:
חֲמִשָּׁה מִי יוֹדֵעַ. חֲמִשָּׁה אֲנִי יוֹדֵעַ. חֲמִשָּׁה חֲמִשֵּׁי תוֹרָה. אַרְבַּע אֲמָהוֹת. שְׁלֹשָׁה אָבוֹת. שְׁנֵי לַחֹת הַבְּרִית. אָחַד אֱלֹהֵינוּ
שְׁבַשְׁמִים וּבְאֲרָץ:
שֵׁשׁ מִי יוֹדֵעַ. שֵׁשׁ אֲנִי יוֹדֵעַ. שֵׁשׁ סְדְרֵי מִשְׁנָה. חֲמִשָּׁה חֲמִשֵּׁי תוֹרָה. אַרְבַּע אֲמָהוֹת. שְׁלֹשָׁה אָבוֹת. שְׁנֵי לַחֹת
הַבְּרִית. אָחַד אֱלֹהֵינוּ שְׁבַשְׁמִים וּבְאֲרָץ:
שִׁבְעָה מִי יוֹדֵעַ. שִׁבְעָה אֲנִי יוֹדֵעַ. שִׁבְעָה יְמֵי שַׁבָּת. שֵׁשׁ סְדְרֵי מִשְׁנָה. חֲמִשָּׁה חֲמִשֵּׁי תוֹרָה. אַרְבַּע אֲמָהוֹת. שְׁלֹשָׁה
אָבוֹת. שְׁנֵי לַחֹת הַבְּרִית. אָחַד אֱלֹהֵינוּ שְׁבַשְׁמִים וּבְאֲרָץ:
שְׁמוֹנָה מִי יוֹדֵעַ. שְׁמוֹנָה אֲנִי יוֹדֵעַ. שְׁמוֹנָה יְמֵי מִילָה. שִׁבְעָה יְמֵי שַׁבָּת. שֵׁשׁ סְדְרֵי מִשְׁנָה. חֲמִשָּׁה חֲמִשֵּׁי תוֹרָה. אַרְבַּע
אֲמָהוֹת. שְׁלֹשָׁה אָבוֹת. שְׁנֵי לַחֹת הַבְּרִית. אָחַד אֱלֹהֵינוּ שְׁבַשְׁמִים וּבְאֲרָץ:
תְּשַׁעָה מִי יוֹדֵעַ. תְּשַׁעָה אֲנִי יוֹדֵעַ. תְּשַׁעָה יְרַחֵי לְדָה. שְׁמוֹנָה יְמֵי מִילָה. שִׁבְעָה יְמֵי שַׁבָּת. שֵׁשׁ סְדְרֵי מִשְׁנָה. חֲמִשָּׁה
חֲמִשֵּׁי תוֹרָה. אַרְבַּע אֲמָהוֹת. שְׁלֹשָׁה אָבוֹת. שְׁנֵי לַחֹת הַבְּרִית. אָחַד אֱלֹהֵינוּ שְׁבַשְׁמִים וּבְאֲרָץ:
עֶשְׂרֵה מִי יוֹדֵעַ. עֶשְׂרֵה אֲנִי יוֹדֵעַ. עֶשְׂרֵה דְבָרִים. תְּשַׁעָה יְרַחֵי לְדָה. שְׁמוֹנָה יְמֵי מִילָה. שִׁבְעָה יְמֵי שַׁבָּת. שֵׁשׁ סְדְרֵי
מִשְׁנָה. חֲמִשָּׁה חֲמִשֵּׁי תוֹרָה. אַרְבַּע אֲמָהוֹת. שְׁלֹשָׁה אָבוֹת. שְׁנֵי לַחֹת הַבְּרִית. אָחַד אֱלֹהֵינוּ שְׁבַשְׁמִים וּבְאֲרָץ:
אֶחָד עֶשְׂרֵה מִי יוֹדֵעַ. אֶחָד עֶשְׂרֵה אֲנִי יוֹדֵעַ. אֶחָד עֶשְׂרֵה כּוֹכָבִים. עֶשְׂרֵה דְבָרִים. תְּשַׁעָה יְרַחֵי לְדָה. שְׁמוֹנָה יְמֵי מִילָה.
שִׁבְעָה יְמֵי שַׁבָּת. שֵׁשׁ סְדְרֵי מִשְׁנָה. חֲמִשָּׁה חֲמִשֵּׁי תוֹרָה. אַרְבַּע אֲמָהוֹת. שְׁלֹשָׁה אָבוֹת. שְׁנֵי לַחֹת הַבְּרִית. אָחַד
אֱלֹהֵינוּ שְׁבַשְׁמִים וּבְאֲרָץ:

שָׁנִים עֶשְׂרִי מִיּוֹדֵעַ. שָׁנִים עֶשְׂרִי אֲנִי יוֹדֵעַ. שָׁנִים עֶשְׂרִי שְׁבַעִים. אֶחָד עֶשְׂרִי כּוֹכָבִים. עֶשְׂרִי דְבָרִים. תְּשַׁעַה יְרַחֵי לְדָה. שְׁמוֹנֶה יָמֵי מִלְּהָה. שְׁבַעַה יָמֵי שְׁבַת. שֵׁשָׁה סְדְרֵי מְשֻׁנָּה. חֲמִשָּׁה חֲמִשֵּׁי תוֹרָה. אַרְבַּע אֲמָהוֹת. שְׁלֹשָׁה אָבוֹת. שְׁנֵי לַחֹת הַבְּרִית. אֶחָד אֱלֹהֵינוּ שְׁבַעִים וּבְאָרְצָן:

שְׁלֹשָׁה עֶשְׂרִי מִיּוֹדֵעַ. שְׁלֹשָׁה עֶשְׂרִי אֲנִי יוֹדֵעַ. שְׁלֹשָׁה עֶשְׂרִי מֵדוֹת. שָׁנִים עֶשְׂרִי שְׁבַעִים. אֶחָד עֶשְׂרִי כּוֹכָבִים. עֶשְׂרִי דְבָרִים. תְּשַׁעַה יְרַחֵי לְדָה. שְׁמוֹנֶה יָמֵי מִלְּהָה. שְׁבַעַה יָמֵי שְׁבַת. שֵׁשָׁה סְדְרֵי מְשֻׁנָּה. חֲמִשָּׁה חֲמִשֵּׁי תוֹרָה. אַרְבַּע אֲמָהוֹת. שְׁלֹשָׁה אָבוֹת. שְׁנֵי לַחֹת הַבְּרִית. אֶחָד אֱלֹהֵינוּ שְׁבַעִים וּבְאָרְצָן:

21. Mathematicians debate about whether 1 should be considered a prime number or not, but in the end it is a matter of personal preference. The Torah mindset is of course that 1 is indeed a prime number. So when we say that 37 is the 13th prime, we are counting the primes beginning with and including 1.
22. Exodus ch. 19.
23. Genesis 1:2.
24. *Bava Kama* 17a, *Tana d'vei Eliyahu* 2, 18, *Tana d'vei Eliyahu Zuta* 1. See also, *Tanya* ch. 40.
25. Exodus 16:7-8. Numbers 16:11.
26. See *Likutei Sichot* vol. 10, p. 8, quoting the last day of Pesach 5699 address of the previous Lubavitcher Rebbe.
27. 115 is also the value of the word for “bush” (סִנֵּה) found in God’s first revelation to Moshe at the burning bush (Exodus ???).
- xxviii. 1820 is the product of 10 times 182, the *gematria* of “Jacob” (יַעֲקֹב). The next 4-dimensional triangular number is 2380, which is the product of 10 times 238, the *gematria* of “Rachel” (רַחֵל), Jacob’s wife!
- xxix. By definition, 1820 is the sum of the first 13 tetrahedral (3-dimensional triangular) numbers. If we divide these 13 into the first 10 and then the next 3, we get 715 and 1105. 715 is the exact number of times Havayah appears in Genesis and Deuteronomy (the first and last books of the Pentateuch). 1105 is the exact number of times it appears in Exodus, Leviticus, and Numbers (the middle three books).
Furthermore,

$$1820 = 28 \cdot 65$$

And,

$$715 = 11 \cdot 65 \text{ and } 1105 = 17 \cdot 65$$

But, 11 and 17 are the golden section of 28, a topic we will explore further in part II.