Gal Einai
Revealing the Torah's Inner Dimension

BSD

Mathematics in the Torah

Parashat Mishpatim: The evolution of thievery

Point, line, and area

The eighth of the Ten Commandments is,[1] "You shall not steal" and in the original Hebrew, לֹא תִּגְנֹב. This prohibitive commandment is immediately expounded upon in our parashah, Mishpatim, in the verse, "One who kidnaps a man and sells him, and he was found to have been in his power, shall surely be put to death,"[2] and in the original Hebrew, וְגֹנֵב אִישׁ וּמְכָרוֹ וְנִמְצָא בְיָדוֹ מוֹת יוּמָת. The way the prohibition appeared in the Ten Commandments, it was not clear whether the reference was to stealing an object or kidnapping a person and this verse serves to clarify the question.

In Deuteronomy, the trend to further clarify the prohibition continues and we find another verse that reads, "If a man is found to have kidnapped a fellow Israelite, enslaving him or selling him, that kidnapper shall die; thus you will sweep out evil from your midst" and in the original Hebrew, כִּי־יִמָּצֵא אִישׁ גֹּנֵב נֶפֶשׁ מֵאֶחָיו מִבְּנֵי יִשְׂרָאֵל וְהִתְעַמֶּר־בּוֹ וּמְכָרוֹ וּמֵת הַגַּנָּב הַהוּא וּבִעַרְתָּ הָרָע מִקִּרְבֶּךָ.

These three verses revolve around the same point: stealing, which turns out to be in this case, stealing a person or kidnapping, with each expanding and deepening our understanding of the prohibition at hand. A three stage developmental description of this nature is known in Kabbalah as a point-line-area model. It is also referred to at times as the point-sefirah-partzuf model, or to borrow the most anthropomorphic language used by the Arizal, an idea developing in these three stages would be in an imprengnation-suckling-mindfulness process.

Number of letters

The most obvious quantitative corollary to a developmental process connecting these three verse can be seen in the increasing number of letters. The first verse only has 6 letters, the second has 28, and the third has 66. Moreover, all three numbers are triangular numbers, strongly suggesting that they are indeed related to one another: 6 is the triangle of 3 (sum of integers from 1 to 3), 28 is the triangle of 7 (sum of integers from 1 to 7), and 66 is the triangle of 11 (sum of integers from 1 to 11).

It therefore is fitting to draw the three verses in triangular form, like so:

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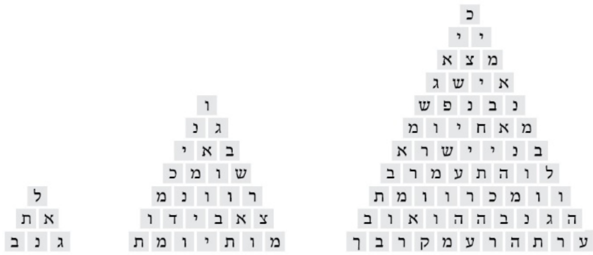
1. Exodus 20:13.
2. Ibid. 21:16.



The next thing we should notice is that the sum total letters in all three verses is 100, which means that the three verses complement one another perfectly and can be drawn in the shape of the square of 10, like so:

Whenever the sum of two quantities (all the more so, three quantities) is a square number, that indicates an essential connection between the two (or more) entities. Looking more closely at the square shape of the three verses, we note that the value of the letters in the diagonal running from top-right to bottom left—**ליאתנבעמבך**—is 625, which itself is not only a square number (25^2), but the square of a square, or 5^4.

Sums of triangles that are squares

Let us try to generalize the beautiful phenomenon just discovered, by asking: Are there any other examples of three triangular numbers whose sum is a square number? Before answering our question into an equation, let us also note that the three indexes of the triangular numbers in our phenomenon are not unrelated. The indexes are 3, 7, and 11, and we immediately note that the difference between each pair is 4. If we generalize this, we can say that the indexes of the three triangular numbers are k, $k + i$, and $k + 2i$. Thus, we are essentially asking, for what n, i, and m is the following general equation with 3 variables fulfilled:

$$\triangle n + \triangle(n + i) + \triangle(n + 2i) = m^2$$

Actually, the simplest approach would be to set i equal to 0 and get,

$$\triangle n + \triangle(n) + \triangle(n) = m^2$$

$$3\triangle n = m^2$$

With trial and error it is easy to find that this equation will be fulfilled when n equals 2, because, 3 times $\triangle 2$ is a square number, 9:

$$3\triangle 3 = 3(3) = 9 = 3^2$$

If we now set i equal to 1, we can once again use trial and error to find that this equation will be fulfilled if we set n equal to 5:

$$\triangle 5 + \triangle 6 + \triangle 7 = m^2$$

$$15 + 21 + 28 = 64 = 8^2$$

If we set i equal to 2, we will find by trial and error that the three numbers are:

$$\triangle 8 + \triangle 10 + \triangle 12 = m^2$$

$$36 + 55 + 78 = 169 = 13^2$$

If we set i equal to 3, we will find by trial and error that the three numbers are:

$$\triangle 11 + \triangle 14 + \triangle 17 = m^2$$

$$66 + 105 + 153 = 169 = 18^2$$

And so on.

Another family of equations can be found by setting n equal to 0 and i equal to 1 getting,

$$\triangle 0 + \triangle(0 + 1) + \triangle(0 + 2) = m^2$$

$$0 + 1 + 3 = 2^2$$

Thus, (0, 1, 2) is another three-tuple solution, from which we can again using trial and error generate another family of solutions. To do so, we note that in the previous family of solutions, (2, 2, 2), (5, 6, 7), (8, 10, 12), (11, 14, 17), the first number in each three-tuple is greater by 3 than the first number in the previous three-tuple and that of course, the difference between the numbers in the three-tuple itself increases by 1 each time.

Doing the same for (0, 1, 2) we find that (3, 5, 7) and (6, 9, 12) and (9, 13, 17) are all solutions as well.

Writing these families of solutions one under the other,

(2, 2, 2)	(5, 6, 7)	(8, 10, 12)	(11, 14, 17)
(0, 1, 2)	(3, 5, 7)	(6, 9, 12)	(9, 13, 17)

We find that the final number in each three-tuple in each family is the same! If we write the squares that they produce one under the other, we get,

$3^2 = 9$	$8^2 = 64$	$13^2 = 169$	$18^2 = 324$
$2^2 = 4$	$7^2 = 49$	$12^2 = 144$	$17^2 = 289$

And we might notice that the index of the square in the first family is 1 more than the largest number in the three-tuple, while in the second family of solutions, it is exactly the same as the largest number in the three-tuple.

A general multiplication rule

Let us return to look at our original three indexes: 3, 7, and 11. Their product is,

$$3 \cdot 7 \cdot 11 = 231$$

But, 231 is itself a triangular number, 231 = $\triangle 21$. Which itself is astounding because 21 is also the sum of 3, 7, and 11, or,

$$\triangle 3 + \triangle 7 + \triangle 11 = 3 \cdot 7 \cdot 11$$

So we have here another phenomenon that can be generalized by asking when else will this be the case. To find the answer, let us again generalize and rerewrite our finding as,

$$\triangle[a + (a + n) + (a + 2n)] = a(a + n)(a + 2n) \qquad \text{; call this equation (1)}$$

Since this equation has 2 variables, to solve we will have to find another equation to describe our 3 numbers. Indeed, we notice that:

$$3 + 7 + 11 = 3 \cdot 7$$

Or in other words, that,

$$a + a + n + a + 2n = a \cdot (a + n)$$

$$3(a + n) = a(a + n)$$

It is easy then to see that this second equation will be satisfied is we set a equal to 3,

$$3(3 + n) = 3(3 + n)$$

We can then go back to equation (1) and solve for n, getting,

$$n^2 - n - 12 = 0$$

Yielding the result, $n = 4$.