

Mathematics in the Torah

Devarim: 3 Verses and 3 Figurate Numbers

Devarim's first two verses

Let's look at the first three verses of the Pentateuch's fifth book, or *chumash*, Deuteronomy (or *Devarim*):

Verse 1: These are the words that Moses addressed to all Israel on the other side of the Jordan, through the wilderness, in the Arabah near Suph, between Paran and Tophel, Laban, Hazereth, and Di-zahab.¹

אלה הדברים אשר דבר משה אל כל ישראל בעבר הירדן במדבר בערבה מול סוף בין פארן ובין תפל ולבן וחצרת ודי זָהָב.

Verses 2 and 3: It is eleven days from Horeb to Kadesh-barne'a by the Mount Seir route. It was in the fortieth year, on the first day of the eleventh month, that Moses addressed the Israelites in accordance with the instructions that God had given him for them.²

אחד עשר יום מחרב דרך הר שעיר עד קדש ברנע. ויהי בארבעים שנה בעשתי עשר חדש באחד לחדש דבר משה אל בני ישראל ככל אשר צוה הוי' אתו אלהם.

Normally, the first thing we do when approaching the mathematical dimension of the Torah is to count the number of letters (and words) in a verse or group of verses. In this case, we find that the first verse has 81 letters, while the next 2 verses (verses 2 and 3) have 100 letters. Both 81 and 100 are square numbers. 81 is the square of 9 and 100 is the square of 10. Notice that 81 and 100 are sequential squares.

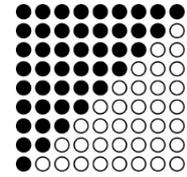
One thing we might start with is then to write the first verse in the form of the square of 9 and the second and third verses in the form of the square of 10. We leave this as an exercise for the interested reader. We would like to delve more deeply into the structure of the first verse.

Almost every verse in the Torah is divided into two parts, marked by the cantillation mark called the *etnachta*. These two parts are not necessarily equal. In our first verse, the *etnachta* divides the verse under the word "the Jordan" (הירדן). Thus, if we count, we find that in the verse's first part (up to and including the word "the Jordan") there are 36 letters (אלה הדברים אשר דבר משה אל כל ישראל בעבר הירדן); the second part then necessarily contains 45 letters (במדבר בערבה מול סוף בין פארן ובין תפל ולבן וחצרת ודי זָהָב). Both 36 and 45 are triangular numbers, meaning that they are the sum of the integer (whole) numbers, from 1 to a certain number n. 36 is the triangle of 8, meaning that it is the sum of the

¹. Deuteronomy 1:1.

². Ibid. 1:2-3.

integers from 1 to 8. 45 is the triangle of 9, meaning that it is the sum of the integers from 1 to 9. For shorthand we use the notation $\triangle 8$ and $\triangle 9$ to designate the triangle of 8 and the triangle of 9. What is special about this division of 81 letters into 36 and 45 is that it demonstrates that every square number is a sum of two sequential triangular numbers. In other words, $9^2 = \triangle 8 + \triangle 9$. This can easily be seen by dividing the figure of the square of 9 into two triangles like so:



There are 45 black circles and 36 white circles. Note that the full figure is a square and that each triangle is a sum of integers, as explained above. This is true for every square number, so we could just write:

$$n^2 = \triangle n + \triangle(n - 1)$$

You can test it for different numbers to see that it works.

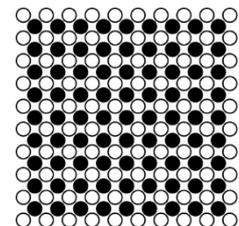
Backtracking a bit, figurate numbers are integers that can be represented in a symmetrical figurate form and that which can be produced by some polynomial function. The three basic figurate numbers we use in the Torah’s mathematical analysis are the triangular numbers, the square numbers, and the interface numbers, which are also referred to as the centered-square numbers. We have just seen how the structure of the first verse of Devarim demonstrates the connection between triangular numbers and square numbers. Now, let us see how all three verses demonstrate the relationship between square numbers and interface numbers.

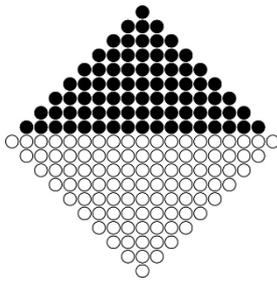
First, let us define an interface number. Every interface number is the sum of two consecutive square numbers. Thus, 1^2 and 2^2 equal 5 and 5 is the interface number of 2. 2^2 and 3^2 equal 13, and so we say that 13 is the interface number of 3, and so on. The general formula for an interface (or center-squared number) is:

$$\square n = n^2 + (n - 1)^2$$

We use the symbol \square to designate an interface number (this is our own nomenclature, it is not readily found in other mathematical works).

Now recall that the first verse had 81 or 9^2 letters and the second and third verses together had 100 letters, which is 10^2 . Since 9^2 and 10^2 are sequential square numbers, their sum, 181, is the interface number of 10, or $\square 10$. How would we draw an interface number as a figure? The most straightforward way is to simply place the smaller square inside the larger—that is where we get the name “interface,” since the squares look like they are interfaced together. For 181, we would draw the figure on the right.

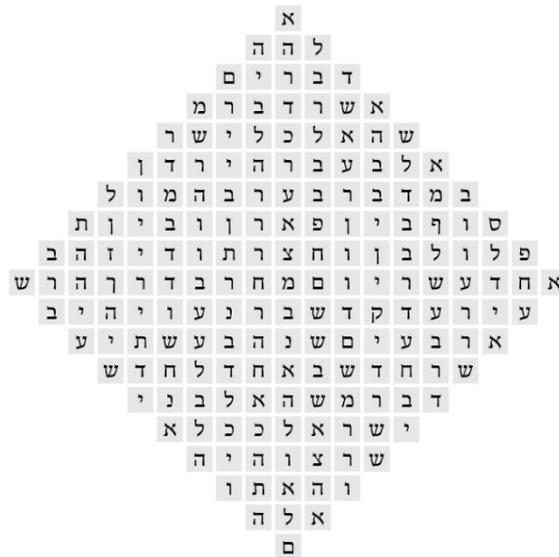




But, by rotating this figure by 45 degrees we get another variation of this figure that actually differentiates between the two squares in another way, as in the figure to the left.

This isomorphic shape is reminiscent of the way that a square number is made up of two triangles, since the two squares are like the two triangular parts of the figure. There are 81 or 9^2 black circles and 100 or 10^2 white circles.

The next figure illustrates how the 181 letters of the three verses would be drawn in this last form,



Notice that the first verse's 81 letters make up the first 9 rows of the figure (81 is 9 squared), while the 100 letters of the second and third verse make up the 10 remaining rows (100 is 10 squared).

Once the verses are depicted in this way, we can find new phenomena. For example, looking at the letters forming the border of the figure, they are: **אהממרו לתבשבעשיאהוהסאושדשאנאפטבאשאל** and there value is 2678, which is the product of 103 and *Havayah*, God's essential Name (יהוה) whose value is 26. 2678 is also the value of the names of the first 10 generations of mankind, "Adam Seth Enosh Keinan Mahalalel Yered Chanoch Methuselah Lemech Noach" (אדם שת אנוש קינן מהללאל ירד חנוך) (מתושלח למך נח). In the same vein, 103 is the value of the first and last of these generations, "Adam Noach" (אדם נח).

Powers of 3

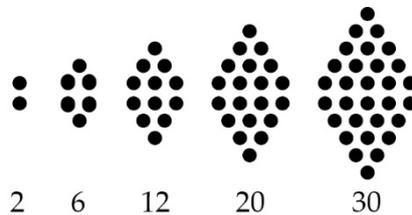
Focusing on the first verse, we find that it has many phenomena related to the powers of 3.

These are the words that Moses addressed to all Israel on the other side of the Jordan, through the wilderness, in the Arabah near Suph, between Paran and between Tophel, Laban, Hatzeroth, and Di-zahab.

אלה הדברים אשר דבר משה אל כל ישראל בעבר הירדן במדבר בערבה מול סוף בין פארן ובין תפל ולבן וחסרת ודי זהב

The first word, "These" (אלה) has 3 letters. Three is the first power of 3, or 3¹. The first full phrase, "These are the words" (אלה הדברים), has 9 or 3² letters. Until the cantillation sign known as a *zakef katan*, marking the end of a larger complete phrase, "These are the words that Moses addressed to all Israel" (אלה הדברים אשר דבר משה אל כל ישראל), there are 27, or 3³ letters. Finally, we have already mentioned that the complete verse has 81, or 3⁴ letters! We might add that the first letter, the *alef* of "These" (אלה) can be seen to represent 3⁰, or a single letter (a further allusion to this is that the numerical value of *alef* is 1).

The entire verse contains 22 words and its complete value is 5060. But, 5060 is amazingly 10 times the diamond number of 22, or 506. A diamond number is simply two identical triangles placed base to base. Here is what they look like,



Mathematicians call the diamond numbers, oblong (or promic) numbers (series A002378 in the oeis.org). The function for these numbers is $\diamond n = 2\Delta n$, or in algebraic form:

$$\diamond n = n(n + 1)$$

Plugging in 22 for n , we get $\diamond 22 = 22(22 + 1) = 506$.

In what is a 1 in 22 chance, the total value of all 22 words, 5060, is a multiple then of 22, which means that each word has an average value: 230.

The sign

We define the sign of a verse as the product of its number of words and its number of letters. The first verse has 22 words and 81 letters, so its sign is 1782. This number is referred to in the unit of the first three verses in another way. The sum of the triangular values of the letters in the first and last words, "These.... for them" (אלה.... אלהם) is:

$$\Delta 1 + \Delta 30 + \Delta 5 + \Delta 1 + \Delta 30 + \Delta 5 + \Delta 40 = 1782$$

One final observation about the first verse, which we saw is intrinsically linked to the number 22 (the number of its words and a factor of its complete value). Let us look at the first and last 22 letters in the verse. The first 22 letters make up the complete first 7 words in the verse, "These are the words that Moses addressed to all" (אלה הדברים אשר דבר משה אל כל). The ratio of 22 to 7 is well-known as the best approximation for pi for two integers smaller than 100. The value of these 7 words is 1430, which is also a multiple of 22, namely 1430 equals 22 times 65, where 65 is the value of God's Name, *Adni* (א-דני). This means that the average value of each letter is *Adni* (א-דני).



The final 22 letters make up the complete last 6 words of the verse, “and between Tophel, Laban, Hatzeroth, and Di-zahab” (ובין תפל ולבן וחצרת ודי זהב). The value of these six words is 1404, which is 54 times *Havayah* (26). 1404 also equals “Shabbat Shabbat” (שבת שבת), alluding to the sages’ statement that were the Jewish people to keep two Shabbatot, they would immediately be redeemed. In reference to the first 22 letters, 1404 is exactly 26, or *Havayah*, less than 1430.